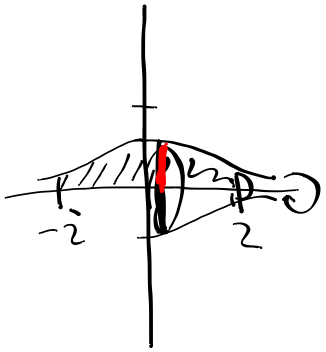
7.2/15 $y = \frac{1}{\sqrt{4+x^2}}$; $x=-2$; $x=+2$; $y=0$



cross-sectional shape?

circle

Area of cross section

$A = \pi r^2$

well then, what is r?

$r = y = \frac{1}{\sqrt{4+x^2}}$

$V = \int_{-2}^{2} \pi \left(\frac{1}{\sqrt{4+x^2}}\right)^2 dx = \pi \int_{-2}^{2} \frac{1}{4+x^2}\, dx$

$= \pi \int_{-2}^{2} \frac{1}{4}\left(\frac{1}{1+\frac{x^2}{4}}\right) dx = \frac{\pi}{4} \int_{-2}^{2} \frac{1}{1+\left(\frac{x}{2}\right)^2} dx$

Let $u = \frac{x}{2}$

$du = \frac{1}{2} dx$

$2\,du = dx$

$\frac{\pi}{4} \int_{-1}^{1} \frac{1}{1+u^2} (2\,du)$

Pattern

$\int \frac{1}{1+u^2} du = \tan^{-1} u$

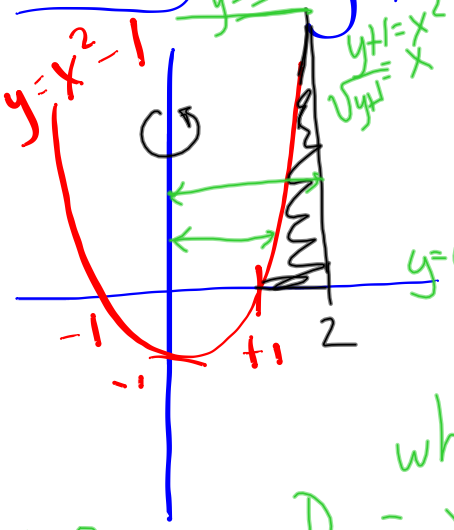
$\text{Volume} = \frac{\pi}{2} \int_{-1}^{1} \frac{1}{1+u^2} du = \frac{\pi}{2} \tan^{-1} u \Big|_{-1}^{1}$

$= \frac{\pi}{2}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right)$

$= \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4}$

Jan 30-7:35 AM

7.2/20 $y = x^2 - 1$; $x = 2$, $y = 0$



cross-section shape?

what are the radiusses?

$y=0$ ⇓ $y=3$

$R_0 = x = 2$

$R_I = x = \sqrt{y+1}$

What is the area of the cross section?

$\pi R_0^2 - \pi R_I^2$

$= \pi (2)^2 - \pi \left(\sqrt{y+1}\right)^2$

$\text{Volume} = \pi \int_0^3 4 - (y+1)\, dy = \pi \int_0^3 3 - y\, dy$

$= \pi \left(3y - \frac{y^2}{2}\right)\Big|_0^3 = \pi \left[\left(9 - \frac{9}{2}\right) - (0)\right]$

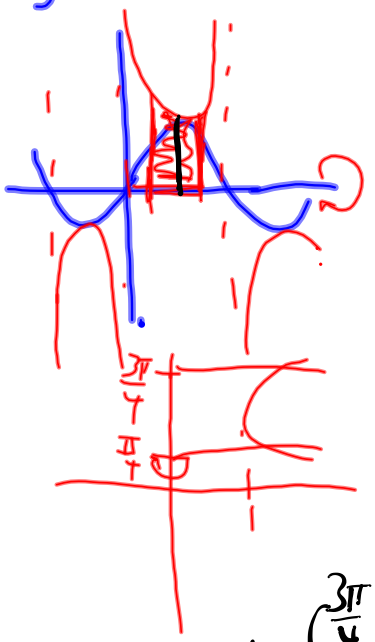
$= \frac{9\pi}{2}$

Jan 30-7:35 AM

7.2/21) $x = \csc y$, $y = \frac{\pi}{4}$, $y = \frac{3\pi}{4}$, $x = 0$

$y = \csc x$ $x = \frac{\pi}{4}$ $x = \frac{3\pi}{4}$

$y = \csc x = \frac{1}{\sin x}$



cross section?

circle

area of X-sec

πr^2

what is r? (if my cheating?)

$r = y = \csc x$

what is volume? (or $= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi (\csc y)^2 \, dy$)

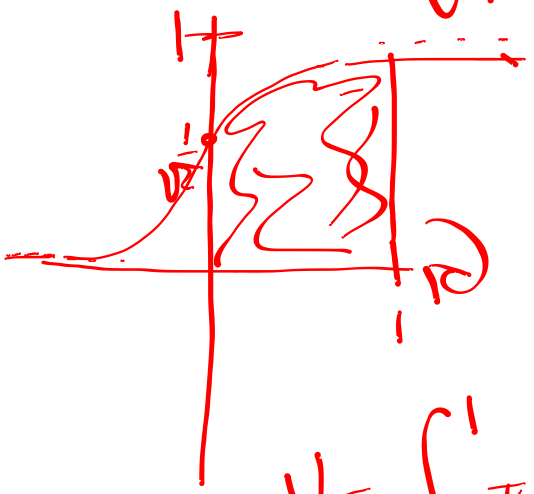
$$V = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi (\csc x)^2 \, dx$$

$$= \pi(-\cot x)\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{-\pi}{\tan x}\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \frac{-\pi}{(-1)} - \left(\frac{-\pi}{1}\right) = 2\pi$$

Jan 30-7:35 AM

7.2/16 $y = \frac{e^{3x}}{\sqrt{1+e^{6x}}}$, $x=0$, $x=1$, $y=0$



- circle
- πr^2
- $\Rightarrow r = y = \frac{e^{3x}}{\sqrt{1+e^{6x}}}$

$$V = \int_0^1 \pi \left(\frac{e^{3x}}{\sqrt{1+e^{6x}}}\right)^2 dx$$

$$= \pi \int_0^1 \frac{e^{6x}}{1+e^{6x}}\,dx$$

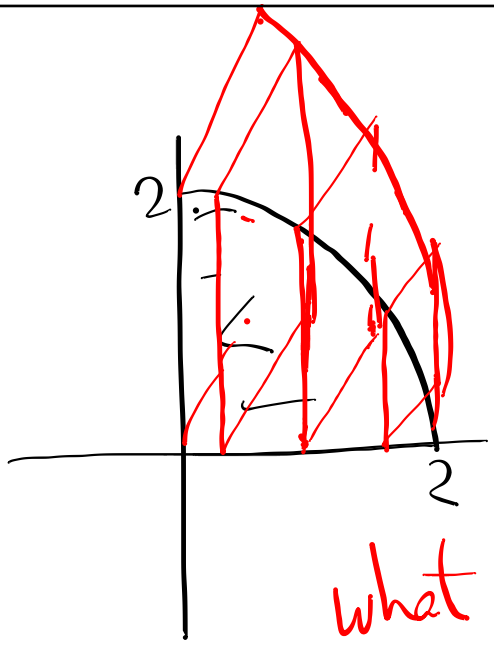
$u = 1+e^{6x}$

$du = 6e^{6x}\,dx$

$$\frac{\pi}{6}\ln(1+e^{6x})\Big|_0^1$$

Find the volume of a solid;
built on the region
bounded by (in Q1)
$y=\sqrt{4-x^2}$; $y=0$; $x=0$
whose cross sections are SQUARES

Jan 30-8:23 AM



what is the X sec shape?

square

area of X sec

S^2 ?

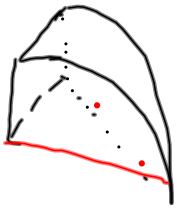
what is s?

$S = y = \sqrt{4 - x^2}$

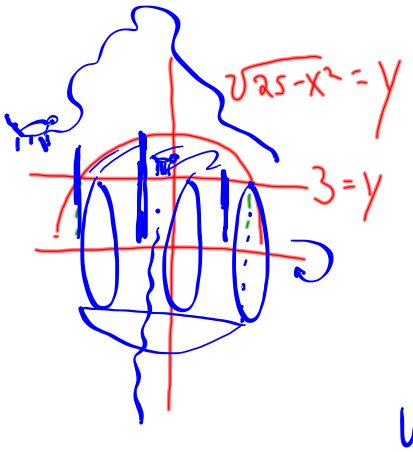
$$\text{Volume} = \int_0^2 \left(\sqrt{4-x^2}\right)^2 dx$$

$$= \int_0^2 4 - x^2 \, dx = \left(4x - \frac{x^3}{3}\right)\Big|_0^2$$

$$= 8 - \frac{8}{3} - 0 = \frac{16}{3}$$



7.2/9



find interactions

$\sqrt{25-x^2}=3$

$25-x^2=9$

$16=x^2 \; ; x=\pm 4$

what is the shape of a cross section?

what is the formula for area of X-sect

$\pi R_O^2 - \pi R_I^2$

what are the radiusss?

$R_O = y = \sqrt{25-x^2}$

$R_I = y = 3$

$$\text{Volume} = \int_{-4}^{4} \pi R_O^2 - \pi R_I^2 \, dx$$

$$\int_{-4}^{4} \pi(\sqrt{25-x^2})^2 - \pi(3)^2 \, dx$$

$$= \pi \int_{-4}^{4} (25-x^2) - 9 \, dx = \pi \int_{-4}^{4} 16 - x^2 \, dx$$

$$= \pi \left(16x - \frac{x^3}{3}\right)\Big|_{-4}^{4}$$

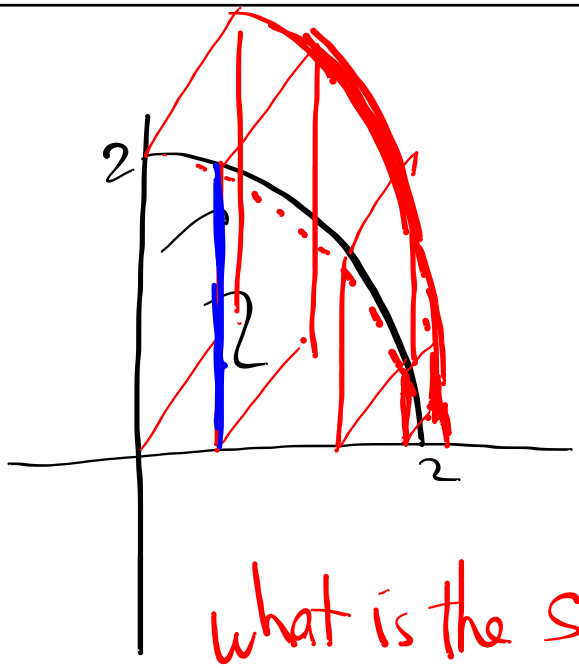
$$= \pi\left(64 - \frac{64}{3} - \left(-64 + \frac{64}{3}\right)\right)$$

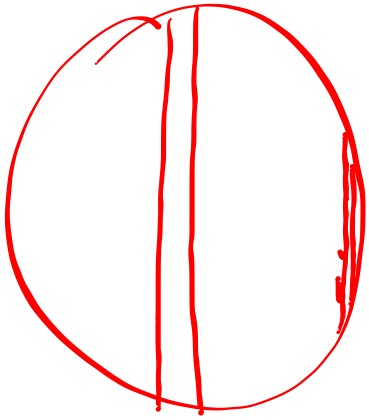
$$= \pi\left(128 - \frac{128}{3}\right) = \frac{256\pi}{3}$$

Jan 30-8:40 AM

Find the volume of a solid, built on the base of a quarter circle [in QI, bounded by $y=\sqrt{4-x^2}$, $x=0$, $y=0$] whose cross-sections are SQUARES.

Jan 30-11:27 AM





what is the shape of a cross-section?

square

what is the area of a square?

what is s? s^2

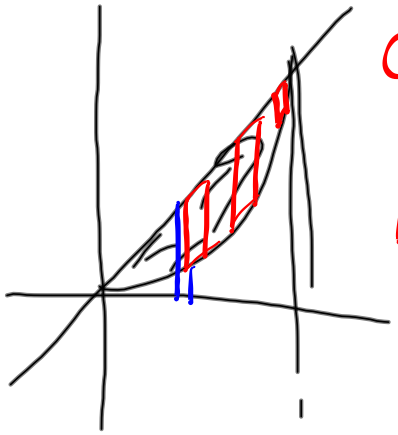
$s = y = \sqrt{4-x^2}$

$$\text{Volume} = \int_0^2 \underbrace{\left(\sqrt{4-x^2}\right)^2}_{s^2} dx = \int_0^2 4-x^2\,dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Big|_0^2 = 8 - \frac{8}{3} - 0 = \frac{16}{3}$$

72/37 base: region bounded by $y=x$ and $y=x^2$

cross sections are squares



cross-section: squares

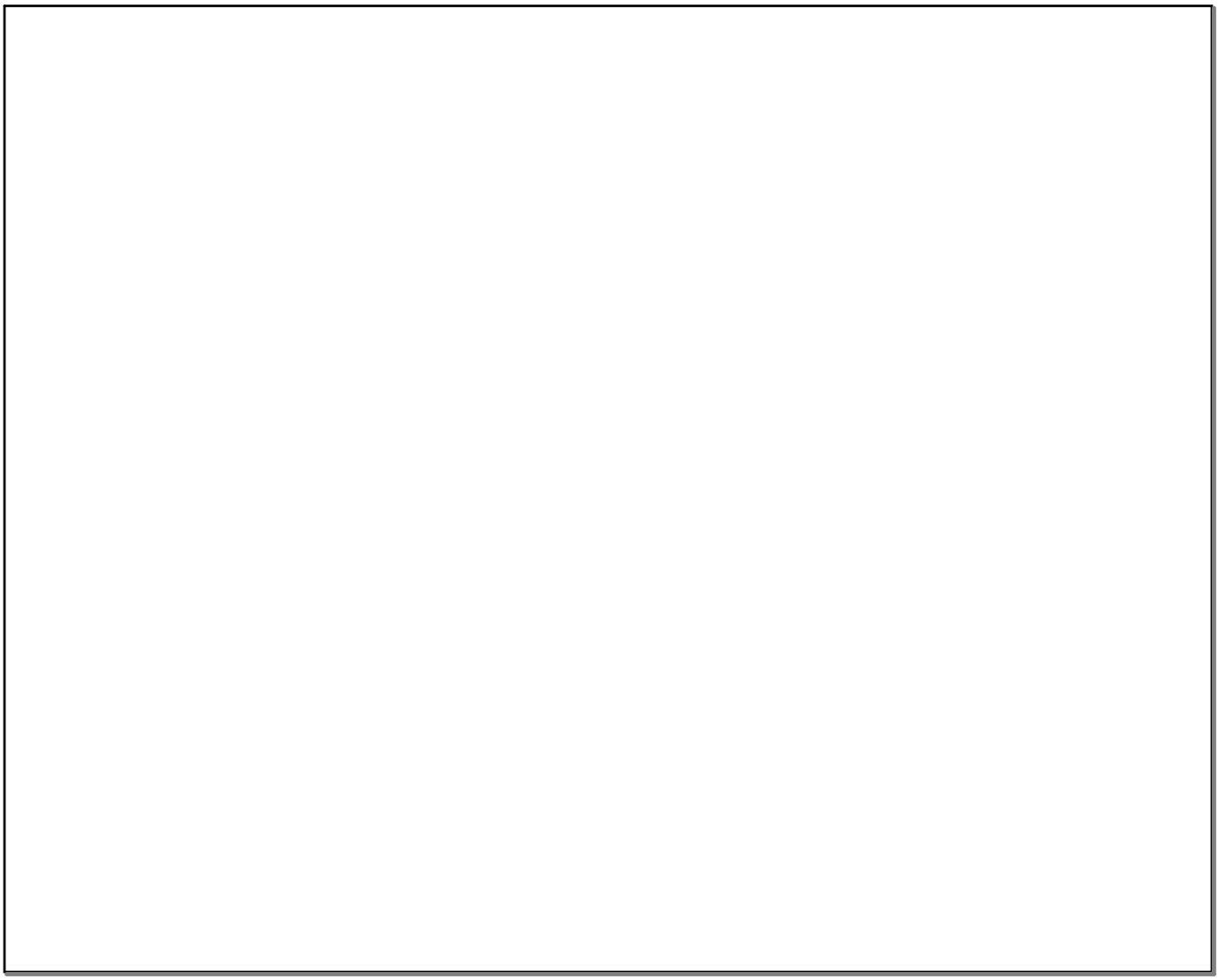
$A_{cross-sect}$: area $= s^2$

what is s?

$s = y_T - y_L$

$= (x) - (x^2)$

Jan 30-11:27 AM



Jan 30-2:53 PM